

**'Jagiellonian Compromise'
- an alternative voting proposal
for the Council of the EU**

Wojciech Słomczyński

Karol Życzkowski

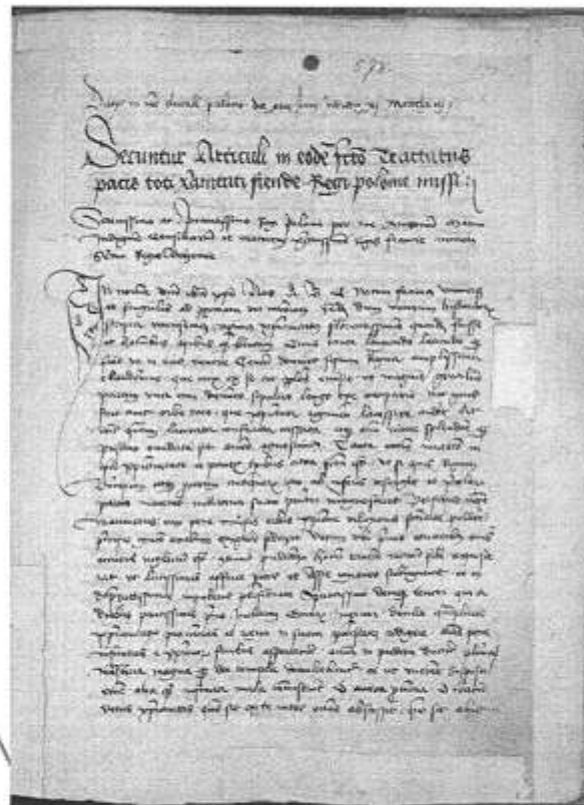
Jagiellonian University

(Kraków)

Jiří z Poděbrad [George of Poděbrady] (1461-1464)

Traité d'alliance et confédération entre le Roy Louis XI. George Roy de Bohême et la Seigneurie de Venise, pour resister au Turc

[Treaty of Alliance and Confederation between King Louis XI, George King of Bohemia and the Seignior of Venice, to resist the Turk]



Rex Franciae, cum coeteris Regibus et Principibus Galliae	1
Reges et Principes Germaniae	1
Dux Venetorum, cum Principibus communitatibus Italiae	1
Rex Castellae, vel alii nationis Hispaniae Reges et Principes	1
TOTAL	4
MAJORITY QUOTA	75%

William Penn (1693)

Essay on the Present and Future Peace of Europe



(38)
pire d'Allemagne envoye douze Députez , la France dix , l'Espagne dix , l'Italie huit , l'Angleterre six , le Portugal trois , la Suède quatre , le Danemarck trois , la Pologne quatre , Venise trois , les sept Provinces quatre , les Cantons Suisses & leurs Alliez deux , les Ducs de Holstein & de Curlande un , & si les Turcs & les Moscovites vouloient entrer dans ce Projet , comme il seroit juste , dix chacun , cela seroit en tout quatrevingt-dix ; Assemblée nombreuse à la vérité , mais non pas trop si l'on considère qu'elle doit représenter la quatrième & la meilleure partie du Monde , aussi-bien que la plus florissante , où la Religion , les Sciences , la Civilité & les Arts ont établi leur sejour. Il ne seroit pas nécessaire

German Empire	12
France	10
Spain	10
Italy	8
England	6
Sweden	4
Poland	4
Netherlands	4
Portugal	3
Denmark	3
Venice	3
Switzerland	2
Holstein/Courland	1
TOTAL	70
QUOTA	75%

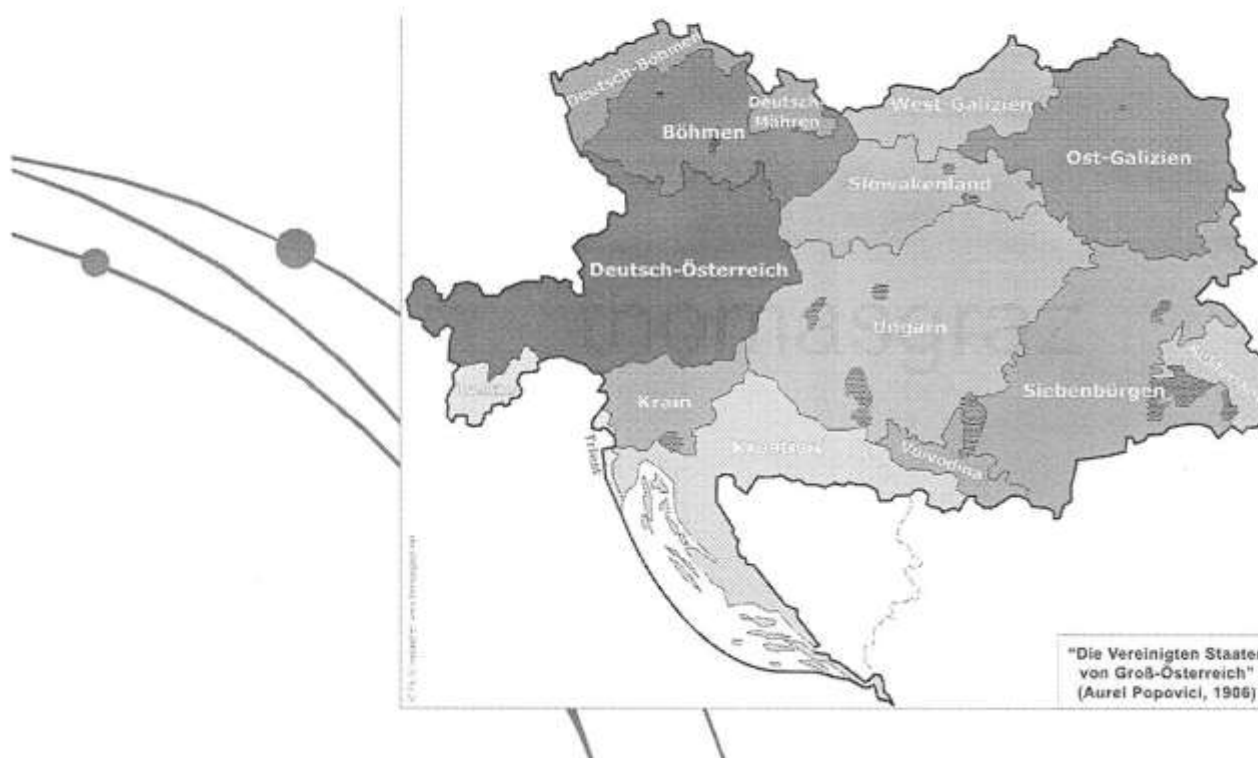
Aurel C. Popovici (1906)

Die Vereinigten Staaten von Gross-Österreich

[The United States of Greater Austria]



German Austria	7
Hungary	7
Bohemia	5
Transylvania	4
Croatia	3
Western Galicia	3
Eastern Galicia	3
German Bohemia	2
Slovakia	2
German Moravia	2
Carniola	1
Szeklerland	1
Trentino	1
Trieste	1
Vojvodina	1
TOTAL	42
QUOTA	> 50%





How to analyse and design voting systems?

- **voting power** (capacity to affect decisions)
- **voting weight** (number of votes)
- **voting power** held by a given state depends not only on its **voting weight** but also on the distribution of the **weights** among all the remaining states
- the **voting power** needs not to be proportional to the **voting weight**

*‘(...) to allocate the **voting weights** on the basis of general philosophical principles that can be seen to apply equally to all countries and citizens’.*
[Leech 2003]

How to measure voting power?

- **power index** - probability that the vote of a country will be decisive in a hypothetical ballot measures the potential voting power

natural normative assumption:

all potential results of voting are equally likely

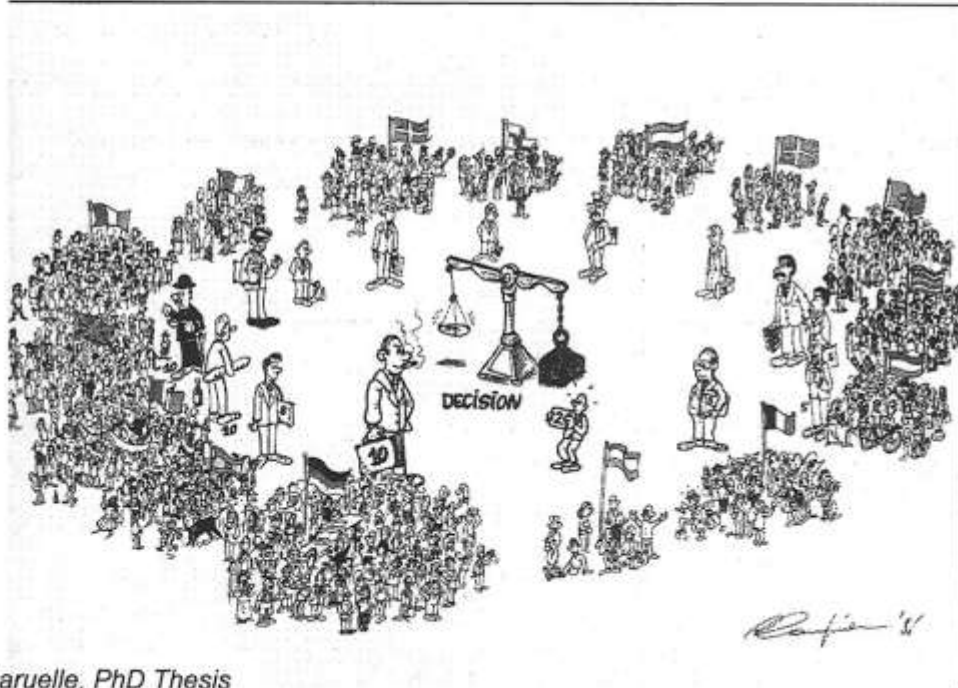


Penrose-Banzhaf index




Voting systems for the Council of the European Union

- a two-tier decision-making system:
 - the **Member States** at the lower level
 - the **European Union** at the upper level
- **27** Members States: more than **134 mln** possible coalitions



The Council of the EU
votes by a **qualified
majority voting**:
a decision of the Council
is taken, if it is approved
by a qualified majority



■ Is it possible to objectively design a voting system, in which each citizen of each Member State of the EU would have the same power to influence the decisions made on his or her behalf?

■ Can it be done in a way that is simple, transparent, easy to implement, efficient to use, and will readily accommodate any future extensions of the EU?

We (still) believe that the answer is

YES



How to allocate voting weights?

- *'one country-one vote'* ? **WRONG**

as if the European Union were a loose association of states

- *the 'Founding Fathers' of Europe explicitly rejected 'objective' keys and population, in favour of a distribution of votes reflecting a balancing act between states. This balance responded to a general principle of 'degressive proportionality' by which the larger units are under-represented compared to the smaller ones [Best 2004].*

- *weights proportional to population* ? **WRONG**

as if the European Union were a single state

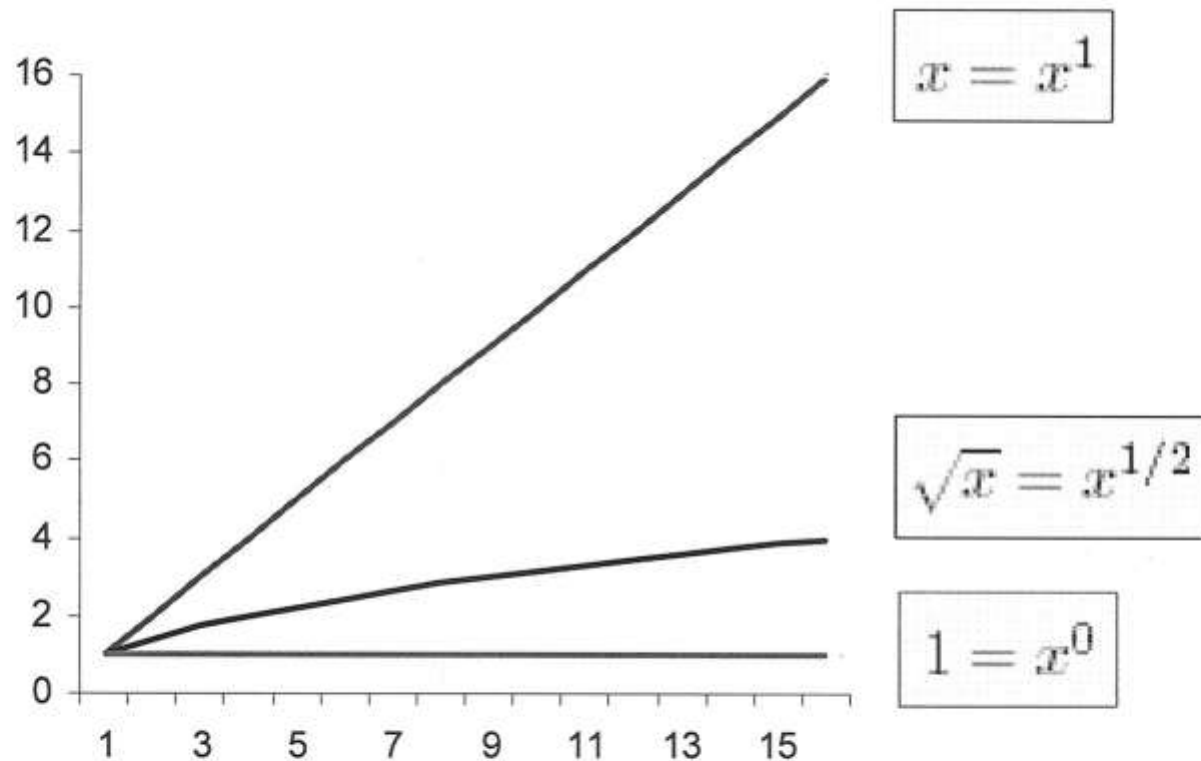
*‘I cannot conceive of the Community
without total parity’*



*Konrad Adenauer to Jean Monnet at a meeting
held in Bonn on 4th April 1951
during the preparations leading up to the signing of the
‘Treaty Establishing the European Coal and Steel Community’*

Square root

square root is the simplest mathematical implementation of the degressive proportionality



Square root weights - example



- the 'square root' weights attributed to Member States are proportional to the sides of the squares representing their populations



Square root weights - support from politicians



- **Göran Persson (2000):** *Our formula has the advantage of being **easy to understand by public opinion** and **practical to use in an enlarged Europe** [...] it is **transparent, logical and loyal**.*
- **John Bruton (2004):** *Instead of double majority, we could put in the Treaty a new, **clear and automatic** mathematical formula for allocating voting weights. Such a formula has been proposed by researchers in the London School of Economics. Their formula would allocate voting weights to countries on the basis of the square root of their population, rather than the number of population itself.*



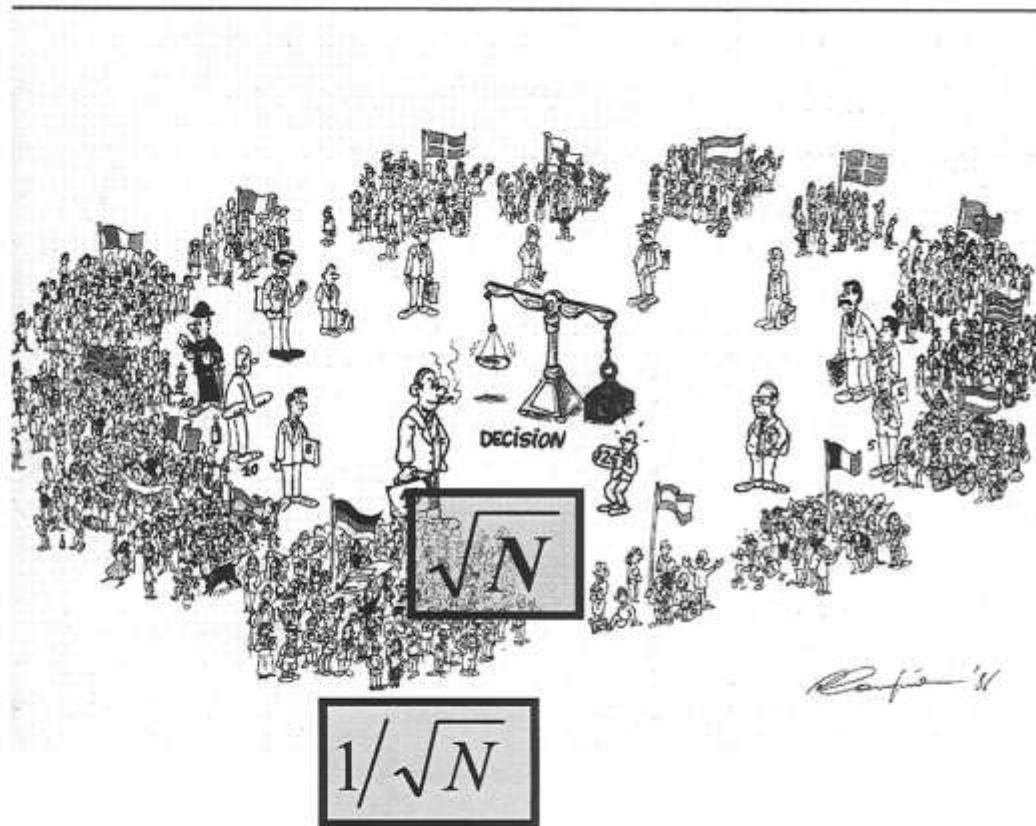
Square root weights

- support from academics

- advocated or analysed by Laruelle, Widgrén (1998), Felsenthal, Machover (1998-2007), Baldwin, Berglöf, Giavazzi, Widgrén (2000), Hosli (2000), Sutter (2000), Tiilikainen, Widgrén (2000), Kandogan (2001), Leech (2002), Moberg (2002, 2007), Hosli, Machover (2002), Leech, Machover (2003), Widgrén (2003), Baldwin, Widgrén (2004), Bilbao (2004), Bobay (2004), Kirsch (2004), Lindner (2004), Lindner, Machover (2004), Plechanovová (2004, 2006), Sozański (2004), Życzkowski, Słomczyński (2004), Ade (2005), Koornwinder (2005), Pajala (2005), Maaser, Napel (2006), Słomczyński, Życzkowski (2006, 2007), Taagepera, Hosli (2006), Życzkowski, Słomczyński, Zastawniak (2006), Algaba, Bilbao, Fernández (2007), Feix, Lepelley, Merlin, Rouet (2007), Kirsch (2007), Leech, Aziz (2007), Machover (2007), Pukelsheim (2007)
- prior to the European Union summit in Brussels in June 2004, an *open letter* in support of square-root voting weights in the Council of Ministers endorsed by more than 40 scientists from 10 European countries

Square root weights - Penrose law

- this degressive system is distinguished by the ***Penrose square root law*** (1952)



each citizen of each country has the same potential voting power, if the a priori voting power of each representative in the voting body should behave proportionally to the square root of the number of citizens



How to allocate voting weights?

*'The square root law tells us how the **power** should be distributed among the countries. It is, however not clear at a first glance how to implement it in terms of voting **weights**, as the voting weights do not give the power indices immediately'. [Kirsch 2004]*

■ How to solve the *inverse problem*, i.e., how to allocate **weights** and how to set **quota (threshold)** for qualified majority to obtain required distribution of **power**?

■ Laruelle & Widgrén (1998), Sutter (2000), Leech (2002), Lindner & Machover (2004), Widgrén (2004), Pajala (2005), Aziz, Paterson & Leech (2007) – *Chow Parameters Problem* (1961)

■ **We were looking for simple (constitutional) solution of the problem!**



Solution

**The answer we proposed was
surprisingly simple:**

One should choose the **weights** to be proportional to the required **powers** and then find such an **optimal quota q_*** that would produce the maximally *transparent* system, that is a system under which the **voting power** of each member would be approximately equal to its **voting weight**.

Formula for optimal quota

$$q_* \cong \frac{1}{2} \left(1 + \frac{1}{\sqrt{M}} \right)$$

M - number of states

$$q_* \cong \frac{1}{2} \left(1 + \sqrt{w_1^2 + \dots + w_M^2} \right)$$

w_k - weight of *k*-th state

In particular, for the Penrose voting system

$w_k \sim \sqrt{N_k}$, N_k - population of *k*-th state, we get

$$q_* \cong \frac{1}{2} \left(1 + \frac{\sqrt{N_1 + \dots + N_M}}{\sqrt{N_1} + \dots + \sqrt{N_M}} \right)$$

Optimal quota – the EU Council

1 The critical point

Physics for

Distributing votes fairly between Europe to a European Constitution. Karol Zyczak and Karol Zyczak describe how physicists are helping politicians.

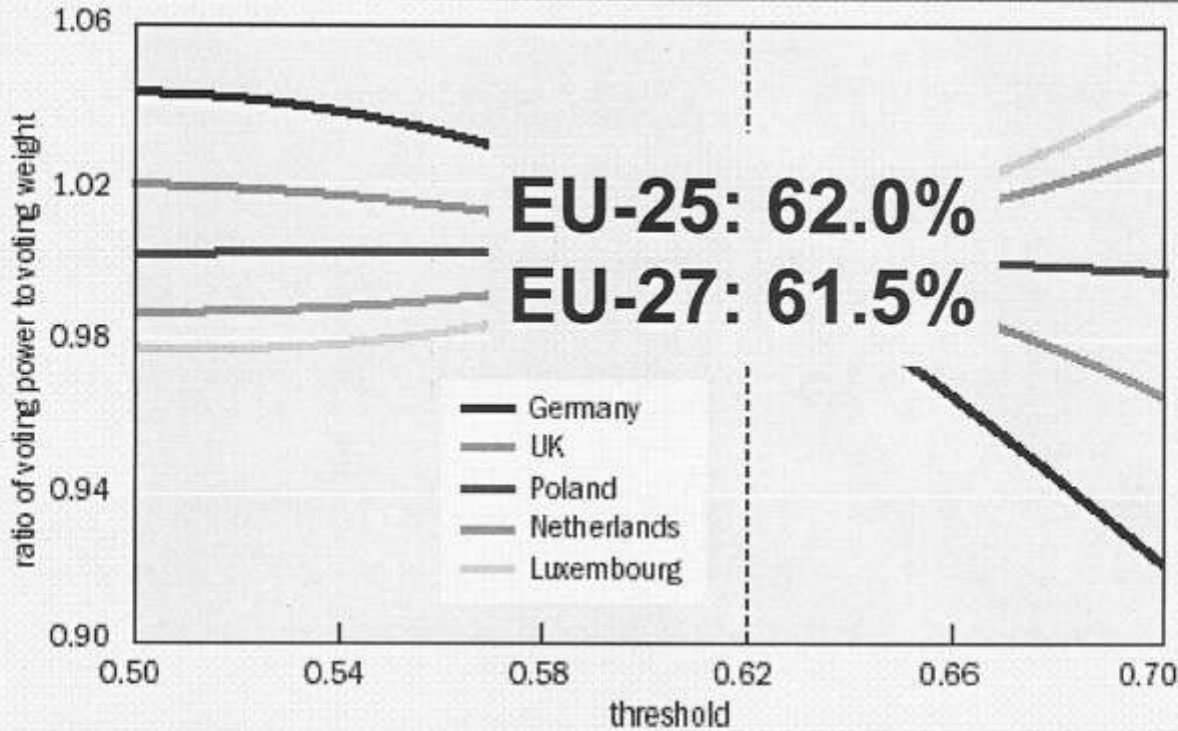
In October 2004 representatives of the Eurozone's 25 member states signed a treaty to establish a European Constitution. Since then the treaty has endured a bumpy ride, with citizens in France and the Netherlands voting against ratification last year. One of the sticking points has been the way votes are distributed between member states in the Council of Ministers, the main decision-making body of the European Union (EU). Physicists and mathematicians are applying their statistical know-how to propose a solution to this problem.

The Council of Ministers consists of politicians from each member state who vote on behalf of their respective countries. To ensure that the influence of each country's vote reflects the size of its population without overwhelming the voice of smaller countries, the treaty's voting rules – set out in 2001 in the Treaty of Nice – are based on a complicated system of “qualified majority voting”. Each country is assigned a voting weight based on its population, and approximately 72% of the total weight must be behind a proposal to be passed. In addition, at least 13 of the 25 states must support the proposal and the population of those states must exceed 62% of the total population of the EU.

During the drafting of the new constitution, politicians decided to simplify this system by dropping voting weights. Instead, decisions would be made by the number of states voting for a proposal and the proportion of the EU population comprised by those states. Under the constitution, a qualified majority would require at least 55% of member states and 60% of the total population to agree.

Although the proposal for the new constitution was simple, it has flaws of its own. Large states, Germany in particular, would gain from the direct link to population, while small countries would derive disproportionate power from the increase in the number of states needed to support a proposal. The combined weight of small states would also increase, while the influence of medium-sized states like Spain and Poland would decrease.

It is possible to objectively design a voting system that addresses these deficiencies, in which each citizen of each member state would have the same power to influence the decisions made on their behalf? Can it be done in a way that is transparent, easy to implement, efficient, and will readily accommodate any future enlargements of the EU, such as the inclusion of Romania and Bulgaria in 2007? The answer, according to research carried out by the present authors, is “yes”.



By plotting the ratio of the voting power of each country to its voting weight against the threshold chosen for majority, a “critical point” emerges. At a threshold of 62% each country achieves a ratio of one, meaning its voting power is equal to its voting weight. Since the weights are chosen to be proportional to the square root of population, the voting power of each citizen is equal. With a lower threshold, larger countries have disproportionate power, while for a higher threshold, smaller countries have more influence.

It is possible to design a voting system in which each citizen of each member state would have the same power to influence decisions made on their behalf?

The sum of weights of the countries voting in favour of a proposal exceeds 62% of the total weight (at $N=25$ and $n=0.62$).

This system has a pleasing simplicity, but how was the magic number of 62% obtained? We first calculated the voting power for each state depends on the threshold value chosen for a majority, and we observed the “critical point”. As the threshold approaches 62%, the voting power of each country, irrespective of its size, approaches the ideal square-root value (see figure 1). The Penrose law is thus fulfilled, and every citizen’s voting power is equalised, with a simpler system than any of the official variations under discussion. Furthermore, any further enlargement of the EU would involve only one recalculation of the threshold for the fixed majority rule – which would become 61.47% for 27 states (EU). Indeed, the critical-point behaviour applies for almost any number of member states or election distribution.

Our proposed voting system has stimulated considerable interest among experts in voting theory, and has been dubbed the “Jagielonian compromise” by the media. Prior to the EU summit in Brussels in June 2004, an open letter in support of square-root voting was presented in the Council of Ministers that was endorsed by more than 40 scientists in 10 European countries sent to EU institutions and the governments of member states.

The reaction of politicians has been varied, but initially depends more on how the Jagielonian compromise affects an individual country’s share of the vote (figure 2) than on universal criteria such as simplicity and objectivity. When a similar system was put forward by Swedish diplomats in 2000, Sweden’s prime minister Goran Persson said, “Our formula has the advantage of being easy to understand by public opinion and practical to use in an enlarged Europe... it is simple, logical and loyal. Maybe that is why it does please everybody.” The former Irish prime minister Bertinotti has also made numerous positive references to voting systems based on Penrose’s law, and Jagielonian compromise has been endorsed by a number of leading politicians in Poland and was seriously considered by UK government researchers in 2004.

It now seems increasingly likely that the European institution fails to come into force, the question of voting in the EU Council of Ministers will be revisited. Jagielonian compromise offers future negotiators a simple but objective system based on rational principles that grants equal voting power to all citizens of the EU.



Jagiellonian Compromise



square root weights

+

optimal quota



Jagiellonian Compromise

- it is extremely **simple** since it is based on a single criterion, and thus it could be called a 'single majority' system;
- it is **objective** (no arbitrary weights or thresholds), hence cannot *a priori* handicap any member of the European Union;
- it is **representative**: every citizen of each Member State has the same potential voting power;
- it is **transparent**: the voting power of each Member State is (approximately) proportional to its voting weight;
- it is **easily extendible**: if the number of Member States changes, all that needs to be done is to set the voting weights according to the square root law and adjust the quota accordingly;
- it is **moderately efficient**: as the number of Member States grows, the efficiency of the system does not decrease;
- it is also **moderately conservative**, that is, it does not lead to a dramatic transfer of voting power relative to the existing arrangements.

*Treaties are like roses and young girls.
They last while they last.*



Charles de Gaulle
Time, 12th July, 1963

